BME I5000: Biomedical Imaging

Lecture 9
Magnetic Resonance Imaging (imaging)

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Blackboard: http://cityonline.ccny.cuny.edu/
Schedule

1. Introduction, Spatial Resolution, Intensity Resolution, Noise
2. X-Ray Imaging, Mammography, Angiography, Fluoroscopy
3. Intensity manipulations: Contrast Enhancement, Histogram Equalisation
4. Computed Tomography
5. Image Reconstruction, Radon & Fourier Transform, Filtered Back Projection
6. Nuclear Imaging, PET and SPECT
7. Maximum Likelihood Reconstruction
8. Magnetic Resonance Imaging
9. Fourier reconstruction, k-space, frequency and phase encoding
10. Optical imaging, Fluorescence, Microscopy, Confocal Imaging
11. Enhancement: Point Spread Function, Filtering, Sharpening, Wiener filter
12. Segmentation: Thresholding, Matched filter, Morphological operations
13. Pattern Recognition: Feature extraction, PCA, Wavelets
14. Pattern Recognition: Bayesian Inference, Linear classification
MRI – How to generate images using NMR

Nuclear spins resonate at a frequency proportional to the external magnetic field

$$\omega = \gamma B_0$$

Basic idea of MRI: Change the $B_0$ field with space and the resonance frequency will change with space.

$$\omega(r) = \gamma B_0(r)$$

The detected resonance signal (FID) contains multiple frequency components each giving information about a different portion of space!
MRI – Signal detected in MRI

Recall that the signal due to the bulk magnetization precessing at $\omega$ detected in the $x$ and $y$ coils can be written as:

$$ s(t) = s_x(t) + i s_y(t) \propto M_{xy}(0) e^{-t/T^*_2} e^{-i \omega t} $$

Signal intensity scales with $M_{xy}(0)$ - the magnitude of the transverse magnetization at the end of the RF pulse. $M_{xy}(0)$ is proportional to the number of resonating spins in the material, or the proton density $\rho(r)$. It is dependent on the tissue and therefore dependent on space $r$.

MRI generates images of $\rho(r)$!

$M_{xy}(0)$ also depends on the specifics of the pulse sequence. By manipulating the pulse sequence MRI can generate images of $\rho(r)$ that are modulated by physical properties that affect $T_1$ or $T_2$. 
MRI – Signal detected in MRI

The main idea is to apply a $B_0$ field with a magnitude that also depends on space, so that the frequency of the resonance signal relates to space, $\omega(r) = \gamma B_0(r)$:

$$s(t) \propto e^{-t/T^*_2} \rho(r) e^{-i \gamma B_0(r)t}$$

(where we have ignored the effect of $T_1$ and $T_2$). The signal emitted by the entire body is then the sum over space:

$$s(t) \propto e^{-t/T^*_2} \int_{\text{body}} d\mathbf{r} \rho(r) e^{-i \gamma B_0(r)t}$$

Note that $B_0(r)$ is parallel to the $z$-axis, only its magnitude may now depend on the location in space $\mathbf{r}$. 
MRI – Signal detected in MRI

For reconstruction it will be useful to define new signal that is 'demodulated' and without the $T_2^*$ decay:

$$S(t) = s(t) e^{t/T_2^*} e^{i \omega_0 t}$$

Define also $\Delta B_z(r)$ as the difference of $B_0(r)$ over main $B_0$:

$$\Delta B_z(r) = B_0(r) - \omega_0 / \gamma$$

With this the MRI imaging equations becomes

$$S(t) = \int_{\text{body}} d\mathbf{r} \rho(\mathbf{r}) e^{-i \gamma \Delta B(r)t}$$
MRI – $B_0$ gradient, frequency encoding

Lets assume we need spatial resolution in only one direction. For instance $x$. So we want to recover (ignoring $z$ direction for now):

$$ g(x) = \int dy \, \rho(x, y) $$

To do so, we apply a contribution $B_0$ that changes linearly with $x$. The strengths of these 'x-gradient' is given by the constants $G_x$.

$$ \Delta B_z(r) = G_x \Delta B_z $$
MRI – $B_0$ gradient, frequency encoding

The imaging equation is now

$$S(t) = \int d\mathbf{x} \, g(x) \, e^{-i \gamma G_x x t}$$

To put this in a more familiar notation lets define a new variable

$$k_x = \gamma G_x t \quad \gamma = \gamma / 2\pi$$

$$S(k_x) = \int d\mathbf{x} \, g(x) \, e^{-i 2\pi k_x x}$$

Evidently the detected signal $S(k)$ is a Fourier transform of $g(x)$, and we can recover it with the inverse Fourier transform.

$$g(x) = \int d\mathbf{x} \, S(k_x) \, e^{i 2\pi k_x x}$$

This methods is therefore called frequency encoding. Obviously we can also apply a $G_y$ gradient and obtain $g(y)$. 
MRI – Axial Reconstruction

Notice that $g(x)$ is the sum of $\rho(x,y)$ along direction $y$, i.e. the direction orthogonal to the gradient $G_x$.

The signal $S(k_x)$ detected in MRI during a $G_x$ gradient is the Fourier transform of $g(x)$.
MRI – Axial Reconstruction

Obviously one can make the gradient $G_{\phi}$ have any orientation $\phi$, and measure the corresponding signal $S(k,\phi)$.

The signal $S(k,\phi)$ detected in MRI during a $G_{\phi}$ gradient is the Fourier transform of the Radon transform $g(r,\phi)$. If we record the signal repeatedly at different orientation $\phi$ we can therefore apply the same Radon reconstruction as in CAT!
MRI – Axial Reconstruction

By combining $x,y$ gradients linearly we can get gradients that at an arbitrary orientation $\phi$:

$$\Delta B_z(r) = G_x x + G_y y = G_\phi \cdot r$$

$$G_\phi = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = G_\phi \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix} \quad r = \begin{bmatrix} x \\ y \end{bmatrix}$$

The signal we obtain is then a Fourier transform of $\rho(r)$ along that direction (the orthogonal directions are summed).

$$S(t, \phi) = \int d\mathbf{r} \rho(\mathbf{r}) e^{\frac{-i}{\gamma} G_\phi \cdot \mathbf{r} t}$$

$$k_\phi = \gamma G_\phi t$$

$$S(k, \phi) = \int d\mathbf{r} \rho(\mathbf{r}) e^{\frac{-i 2\pi}{\gamma} k_\phi \cdot \mathbf{r}}$$
MRI – Gradient Echo Sequence

90° and a $G_x$ gradient will also generate a echo sequence. Spins at some locations spin faster than at others due to $G_x$. After sign reversal of $G_x$ the faster spins become the slower ones, and vice versa. The time it takes the spins to catch up (re-phase) is called echo time ($T_E$).

$$T_E \propto \exp \left( -\frac{t}{T_2} \right)$$
MRI – Gradient Echo Sequence

We can obtain different angles with different combinations of $G_x$ and $G_y$:

$G_x$:  $G \cos \phi$

$G_y$:  $G \sin \phi$

The same can be done with at FID or Echo pulse sequence.
MRI – Gradient Echo Sequence

For this pulse sequence the signal we detect is given by

\[ S(t, \phi) = \int d\mathbf{r} \rho(\mathbf{r}) e^{-i y G_\phi \cdot \mathbf{r}(t-T_E)} \]

\[ \mathbf{k}_\phi = y G_\phi (t-T_E) \]

\[ S(k, \phi) = \int d\mathbf{r} \rho(\mathbf{r}) e^{-i 2\pi k_\phi \cdot \mathbf{r}} \]

or in Cartesian coordinates:

\[ S(k_x, k_y) = \int dx dy \rho(x, y) e^{-i 2\pi (k_x x + k_y y)} \]

i.e. \( S(k_x, k_y) \) is the 2D Fourier transform of \( \rho(x, y) \).
MRI – k-space

Lets consider the RF signal we measure. It represents the data in the frequency domain, i.e. the “k-space”.

For a FID signal

\[
k_\phi = \gamma G_\phi t
\]

For an ECHO signal

\[
k_\phi = \gamma G_\phi (t - T_E)
\]

Time starts at \( t=0 \) and is sampled in discrete points \( t = \Delta t \ n \)
MRI – k-space

Signals taken at multiple angles $\phi$ cover the k-space and allow therefore reconstruction (left).

Is there a pulse sequence that can sample the Fourier space evenly as shown on the right so that we can use direct 2D Fourier inverse?
MRI – Phase Encoding

If spins precess at different speeds (due to a variable encoding gradient $G_y$) during a fixed amount of time $T_{pe}$ they will gain a different phase:
MRI – Phase Encoding

A phase encoding echo pulse sequence, which will sample the \( k \)-space along the \( k_x \) axis for different values of \( k_y \) is as follows:

\[
\text{90}^\circ \quad \text{180}^\circ
\]

\[\text{echo pulse for rectangular k-space sampling}\]

\[\text{Frequency encoding}\]

\[\text{Phase encoding}\]
MRI – Phase Encoding

With a phase encoding gradient $G_y$ in $y$ direction and frequency encoding gradient in $G_x$ in $x$ direction the echo signal would be (ignoring the $z$ direction again):

$$S(t, T_{pe}) = \int d\,x\,dy\, \rho(x, y) e^{-i\gamma G_x (t-T_E) + \gamma G_y y T_{pe}}$$

which can be rewritten as a 2D Fourier transform with the following definitions:

$$k_x = \gamma G_x (t-T_E) \quad k_y = \gamma G_y T_{pe}$$

$$S(k_x, k_y) = \int d\,x\,dy\, \rho(x, y) e^{-i2\pi(k_x x + k_y y)}$$
MRI – Slice selection

So far we considered gradients applied after the RF pulse during free precession. A gradient $G_z$ during the RF pulse will select a transversal slice that satisfies the resonance condition: The RF pulse affects the spin precession coherently only if the frequency matches the $B_z$ field. For the rest $M_{xy} = 0$ after α-pulse.

$M_{xy}(0) = 0$

$M_{xy}(0) = M_0 \sin \alpha$

$M_{xy}(0) = 0$

Only this slice will generate a signal!
MRI – Slice selection

Note that a “hard” RF pulse contains high frequency components. It is therefore less selective in space as a “soft” pulse (sinusoid modulated by a sync function - $\sin(\omega_0 t) * \text{sinc}(\omega)$):

![RF pulse and selected slice graphs](image-url)
MRI – a pulse sequence example

Example for a full pulse sequence with gradient echo and the corresponding path in k-space:

Adapted from http://www.ecf.utoronto.ca/apsc/courses/bme595f/notes/
Echos – refocussing of signal

Spin echo:
use a 180 degree pulse to “mirror image”
the spins in the transverse plane
when “fast” regions get ahead in phase,
make them go to the back and catch up

- measure T2
- ideally TE = average T2

Gradient echo:
flip the gradient from negative to positive
make “fast” regions become “slow” and
vice-versa

- measure T2*
- ideally TE ~ average T2*
**MRI – Summary for Magnetic fields**

- **Main Magnet**
  - High, constant, Uniform Field, $B_0$.

- **Gradient Coils**
  - Produce pulsed, linear gradients in this field.
  - $G_x$, $G_y$, & $G_z$

- **RF coils**
  - Transmit: B1 Excites NMR signal (FID).
  - Receive: Senses FID.

Adapted from http://www.ecf.utoronto.ca/apsc/courses/bme595f/notes/
MRI – Contrast properties

- The strength of the NMR signal produced by precessing protons in a tissue depends on:
  - T1, T2 of the tissue.
  - The density of protons in the tissue.
  - Motion of the protons (flow or diffusion).
  - The MRI pulse sequence used.
- In a T1 “weighted” image the pulse sequence is chosen so that T1 has a larger effect than T2.
- Images can also be made to be T1, T2 proton density or flow/diffusion weighted.

Adapted from http://www.ecf.utoronto.ca/apsc/courses/bme595f/notes/
MRI – Contrast, T1, T2

• MRI Contrast is created since different tissues have different T1 and T2.
  • Gray Matter: (ms) T1= 810, T2= 101
  • White Matter: (ms) T1= 680, T2= 92
• Bone and air are invisible.
• Fat and marrow are bright.
• CSF and muscle are dark.
• Blood vessels are bright.
• Gray matter is darker than white matter.