BME I5000: Biomedical Imaging

Lecture 9
Magnetic Resonance Imaging
(imaging)

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Blackboard: http://cityonline.ccny.cuny.edu/

Schedule

1. Introduction, Spatial Resolution, Intensity Resolution, Noise
2. X-Ray Imaging, Mammography, Angiography, Fluoroscopy
3. Intensity manipulations: Contrast Enhancement, Histogram Equalisation
4. Computed Tomography
5. Image Reconstruction, Radon & Fourier Transform, Filtered Back Projection
6. Nuclear Imaging, PET and SPECT
7. Maximum Likelihood Reconstruction
8. Magnetic Resonance Imaging
9. Fourier reconstruction, k-space, frequency and phase encoding
10. Optical imaging, Fluorescence, Microscopy, Confocal Imaging
11. Enhancement: Point Spread Function, Filtering, Sharpening, Wiener filter
12. Segmentation: Thresholding, Matched filter, Morphological operations
13. Pattern Recognition: Feature extraction, PCA, Wavelets
14. Pattern Recognition: Bayesian Inference, Linear classification

MRI – How to generate images using NMR

Nuclear spins resonate at a frequency proportional to the external magnetic field

$$\omega = \gamma B_0$$

Basic idea of MRI: Change the B_0 field with space and the resonance frequency will change with space.

$$\omega(\boldsymbol{r}) = \gamma B_0(\boldsymbol{r})$$

The detected resonance signal (FID) contains multiple frequency components each giving information about a different portion of space!

MRI – Signal detected in MRI

Recall that the signal due to the bulk magnetization precessing at ω detected in the *x* and *y* coils can be written as:

$$s(t) = s_x(t) + i\, s_y(t) \propto M_{xy}(0)\, e^{-t/T_2^*} e^{-i\omega t}$$

Signal intensity scales with $M_{xy}(0)$ - the magnitude of the transverse magnetization at the end of the RF pulse. $M_{xy}(0)$ is proportional to the number of resonating spins in the material, or the proton density $\rho(\boldsymbol{r})$. It is dependent on the tissue and therefore dependent on space $\boldsymbol{r}$.

MRI generates images of $\rho(\boldsymbol{r})$!

$M_{xy}(0)$ also depends on the specifics of the pulse sequence. By manipulating the pulse sequence MRI can generate images of $\rho(\boldsymbol{r})$ that are modulated by physical properties that affect T_1 or T_2.

MRI – Signal detected in MRI

The main idea is to apply a B_0 field with a magnitude that also depends on space, so that the frequency of the resonance signal relates to space, $\omega(\boldsymbol{r}) = \gamma B_0(\boldsymbol{r})$:

$$s(t) \propto e^{-t/T_2^*} \rho(\boldsymbol{r}) e^{-i\gamma B_0(\boldsymbol{r})t}$$

(where we have ignored the effect of T_1 and T_2). The signal emitted by the entire body is then the sum over space:

$$s(t) \propto e^{-t/T_2^*} \int_{body} d\boldsymbol{r}\, \rho(\boldsymbol{r}) e^{-i\gamma B_0(\boldsymbol{r})t}$$

Note that $B_0(\boldsymbol{r})$ is parallel to the z-axis, only its magnitude may now depend on the location in space $\boldsymbol{r}$.

MRI – Signal detected in MRI

For reconstruction it will be useful to define new signal that is 'demodulated' and without the T_2^* decay:

$$S(t) = s(t)\, e^{t/T_2^*} e^{i\omega_0 t}$$

Define also $\Delta B_z(\boldsymbol{r})$ as the difference of $B_0(\boldsymbol{r})$ over main B_0:

$$\Delta B_z(\boldsymbol{r}) = B_0(\boldsymbol{r}) - \omega_0/\gamma$$

With this the MRI *imaging equations* becomes

$$S(t) = \int_{body} d\boldsymbol{r}\, \rho(\boldsymbol{r})\, e^{-i\gamma \Delta B(\boldsymbol{r}) t}$$

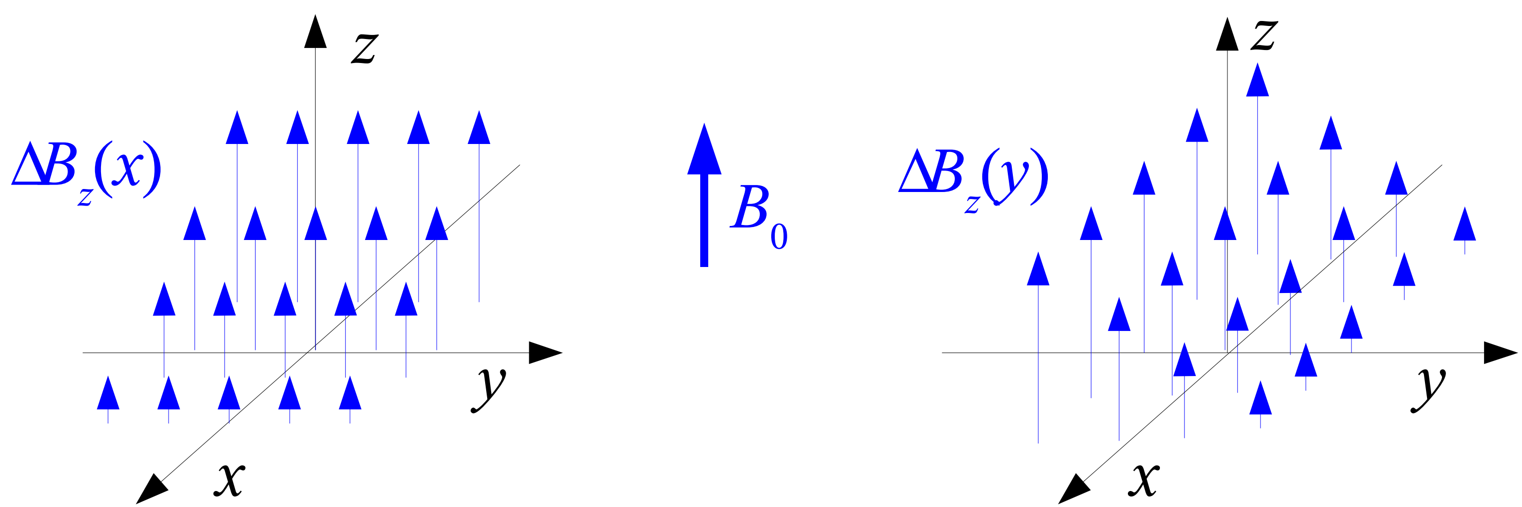
MRI – B_0 gradient, frequency encoding

Lets assume we need spatial resolution in only one direction. For instance x. So we want to recover (ignoring z direction for now):

$$g(x) = \int dy\, \rho(x, y)$$

To do so, we apply a contribution B_0 that changes linearly with x. The strengths of these 'x-gradient' is given by the constants G_x.

$$\Delta B_z(\boldsymbol{r}) = G_x x$$



MRI – B_0 gradient, frequency encoding

The imaging equation is now

$$S(t) = \int dx\, g(x) e^{-i\gamma G_x x t}$$

To put this in a more familiar notation lets define a new variable

$$k_x = \bar{\gamma} G_x t \qquad \bar{\gamma} = \gamma / 2\pi$$

$$S(k_x) = \int dx\, g(x) e^{-i 2\pi k_x x}$$

Evidently the detected signal $S(k)$ is a Fourier transform of $g(x)$, and we can recover it with the inverse Fourier transform.

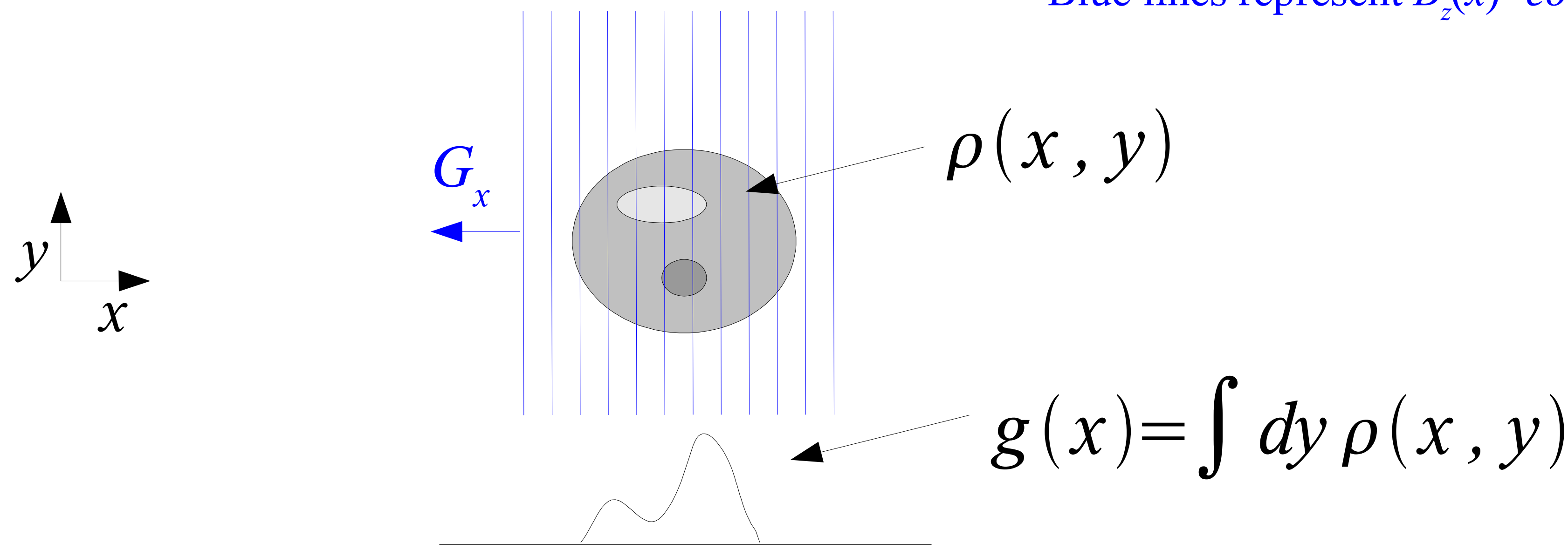
$$g(x) = \int dx\, S(k_x) e^{i 2\pi k_x x}$$

This methods is therefore called ***frequency encoding***. Obviously we can also apply a G_y gradient and obtain $g(y)$.

MRI – Axial Reconstruction

Notice that $g(x)$ is the sum of $\rho(x,y)$ along direction y, i.e. the direction orthogonal to the gradient G_x.

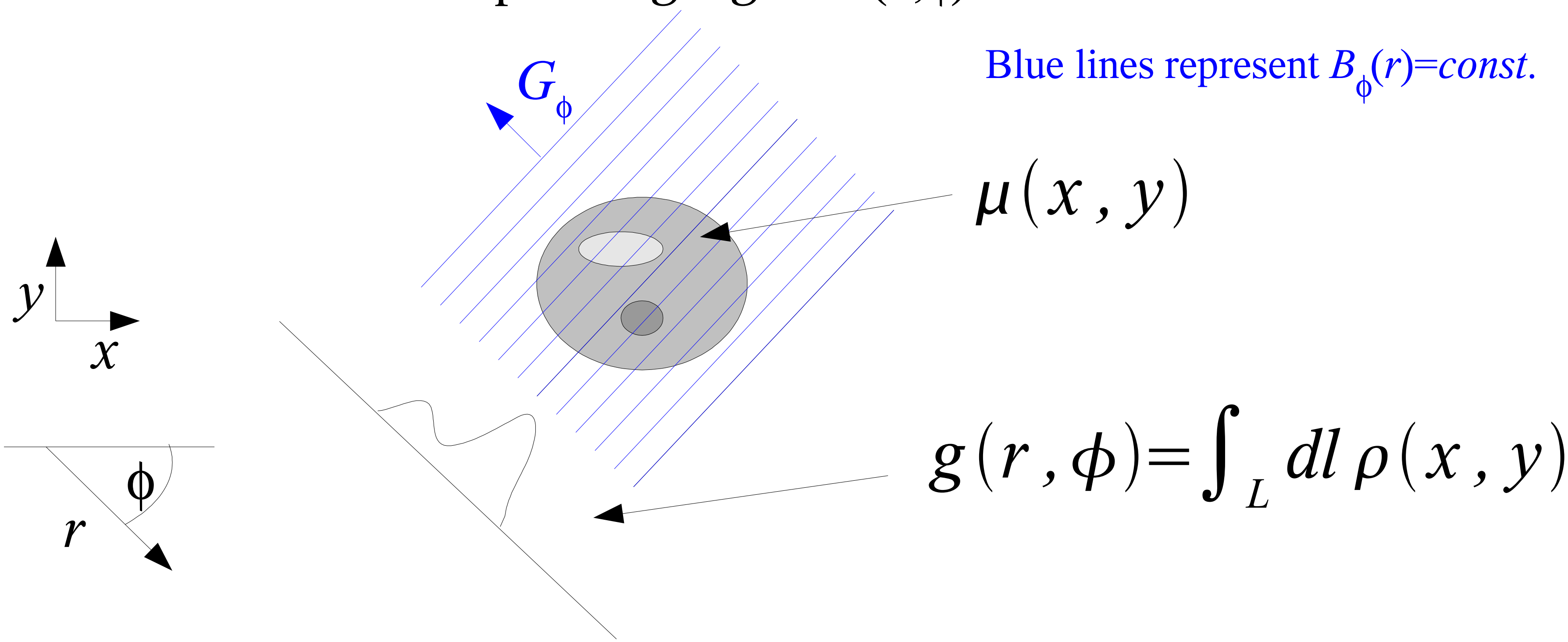
Blue lines represent $B_z(x)=const.$



The signal $S(k_x)$ detected in MRI during a G_x gradient is the Fourier transform of $g(x)$.

MRI – Axial Reconstruction

Obviously one can make the gradient G_ϕ have any orientation ϕ , and measure the corresponding signal $S(k,\phi)$



The signal $S(k,\phi)$ detected in MRI during a G_ϕ gradient is the Fourier transform of the Radon transform $g(r,\phi)$. If we record the signal repeatedly at different orientation ϕ we can therefore apply the same Radon reconstruction as in CAT!

MRI – Axial Reconstruction

By combining *x,y* gradients linearly we can get gradients that at an arbitrary orientation ϕ:

$$\Delta B_z(\boldsymbol{r}) = G_x x + G_y y = \boldsymbol{G}_\phi \cdot \boldsymbol{r}$$

$$\boldsymbol{G}_\phi = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = G_\phi \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix} \qquad \boldsymbol{r} = \begin{bmatrix} x \\ y \end{bmatrix}$$

The signal we obtain is then a Fourier transform of $\rho(\boldsymbol{r})$ along that direction (the orthogonal directions are summed).

$$\boldsymbol{k}_\phi = \begin{bmatrix} k_x \\ k_y \end{bmatrix} = k \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix}$$

$$k = \gamma G_\phi t$$

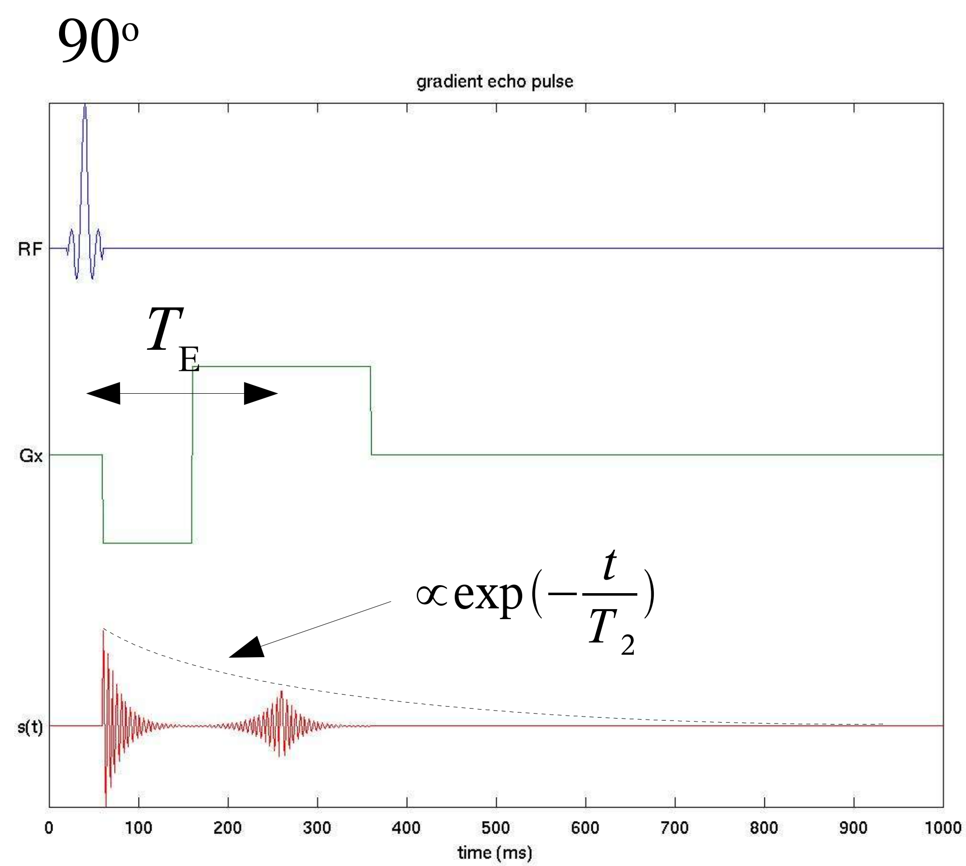
$$S(t,\phi) = \int d\boldsymbol{r}\, \rho(\boldsymbol{r})\, e^{-i\gamma \boldsymbol{G}_\phi \cdot \boldsymbol{r} t}$$

$$\boldsymbol{k}_\phi = \gamma \boldsymbol{G}_\phi t$$

$$S(k,\phi) = \int d\boldsymbol{r}\, \rho(\boldsymbol{r})\, e^{-i 2\pi \boldsymbol{k}_\phi \cdot \boldsymbol{r}}$$

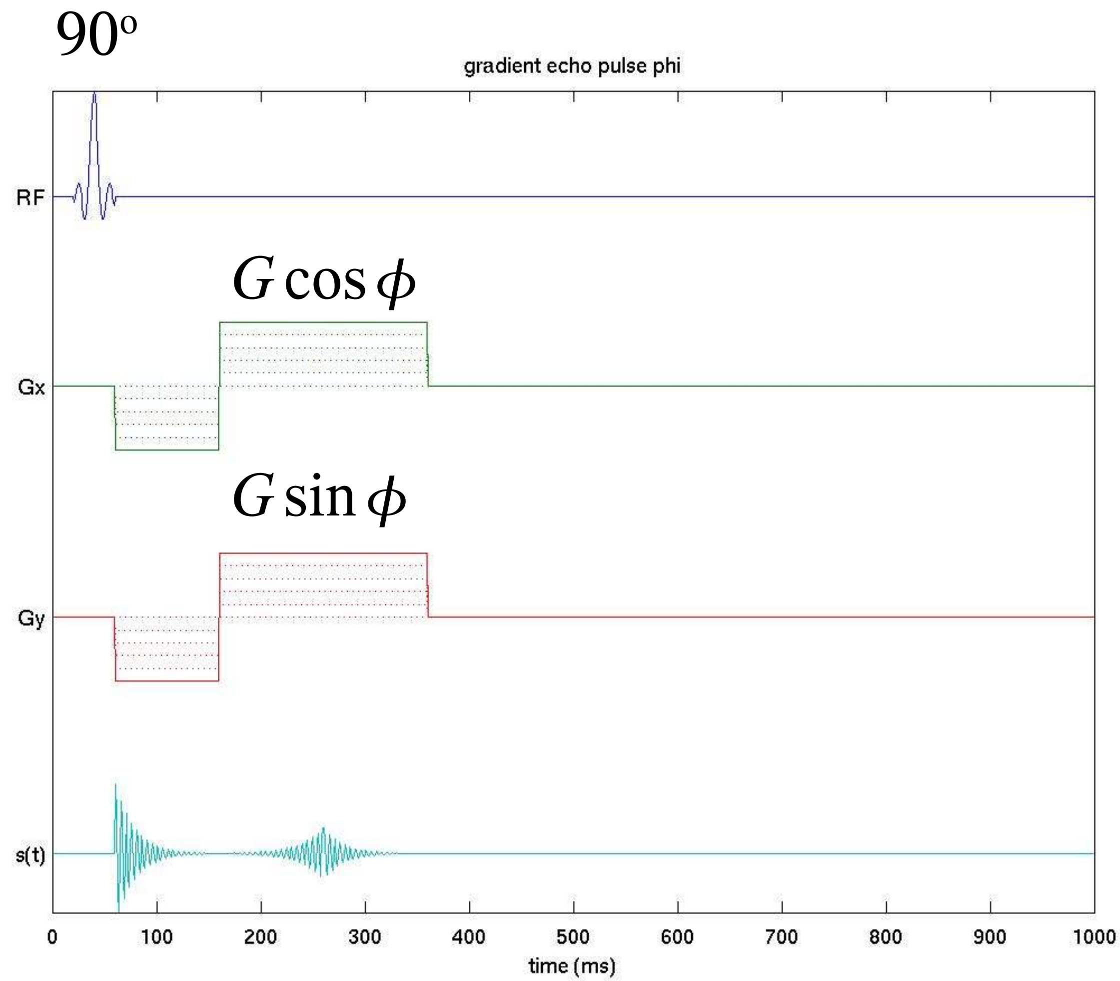
MRI – Gradient Echo Sequence

90° and a G_x gradient will also generate a echo sequence. Spins at some locations spin faster than at others due to Gx. After sign reversal of G_x the faster spins become the slower ones, and vice versa. The time it takes the spins to catch up (re-phase) is called echo time (T_E)



MRI – Gradient Echo Sequence

We can obtain different angles with different combinations of G_x and G_y:



The same can be done with at FID or Echo pulse sequence.

MRI – Gradient Echo Sequence

For this pulse sequence the signal we detect is given by

$$S(t,\phi)=\int d\boldsymbol{r}\,\rho(\boldsymbol{r})\,e^{-i\gamma \boldsymbol{G}_{\phi}\cdot\boldsymbol{r}(t-T_E)}$$

$$\boldsymbol{k}_{\phi}=\gamma\,\boldsymbol{G}_{\phi}(t-T_E)$$

$$S(k,\phi)=\int d\boldsymbol{r}\,\rho(\boldsymbol{r})\,e^{-i2\pi \boldsymbol{k}_{\phi}\cdot\boldsymbol{r}}$$

or in Cartesian coordinates:

$$S(k_x,k_y)=\int dx\,dy\,\rho(x,y)\,e^{-i2\pi(k_x x+k_y y)}$$

i.e. $S(k_x,k_y)$ is the 2D Fourier transform of $\rho(x, y)$.

MRI – k-space

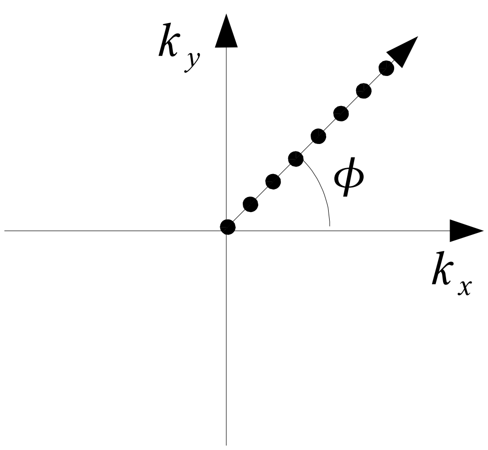
Lets consider the RF signal we measure. It represents the data in the frequency domain, i.e. the "k-space".

For a FID signal

$$\boldsymbol{k}_\phi = \gamma \boldsymbol{G}_\phi t$$

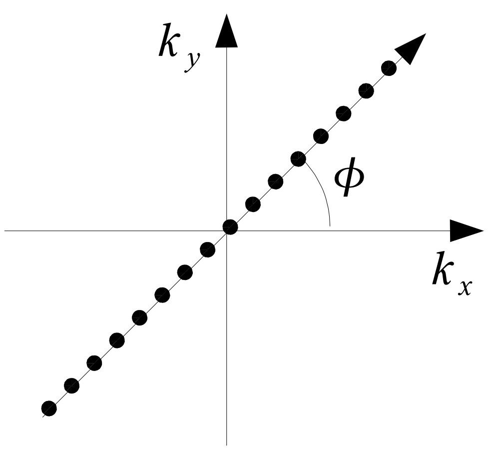
For an ECHO signal

$$\boldsymbol{k}_\phi = \gamma \boldsymbol{G}_\phi (t - T_E)$$



$$\boldsymbol{k}_\phi = \begin{bmatrix} k_x \\ k_y \end{bmatrix} = k \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix}$$

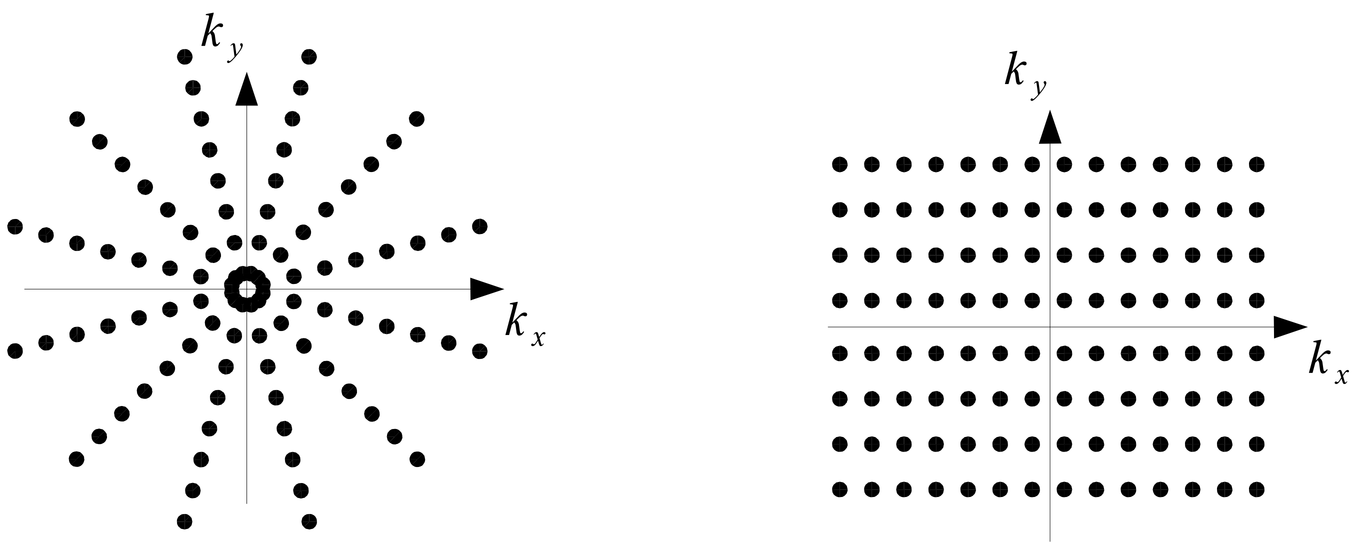
$$\boldsymbol{G}_\phi = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = G_\phi \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix}$$



Time starts at t=0 and is sampled in discrete points $t = \Delta t \, n$

MRI – k-space

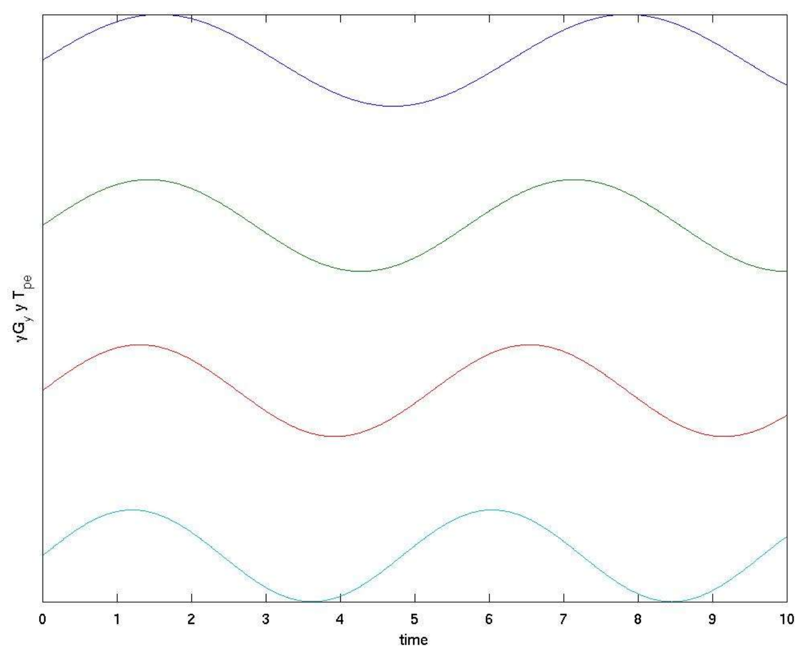
Signals taken at multiple angles ϕ cover the k-space and allow therefore reconstruction (left).



Is there a pulse sequence that can sample the Fourier space evenly as shown on the right so that we can use direct 2D Fourier inverse?

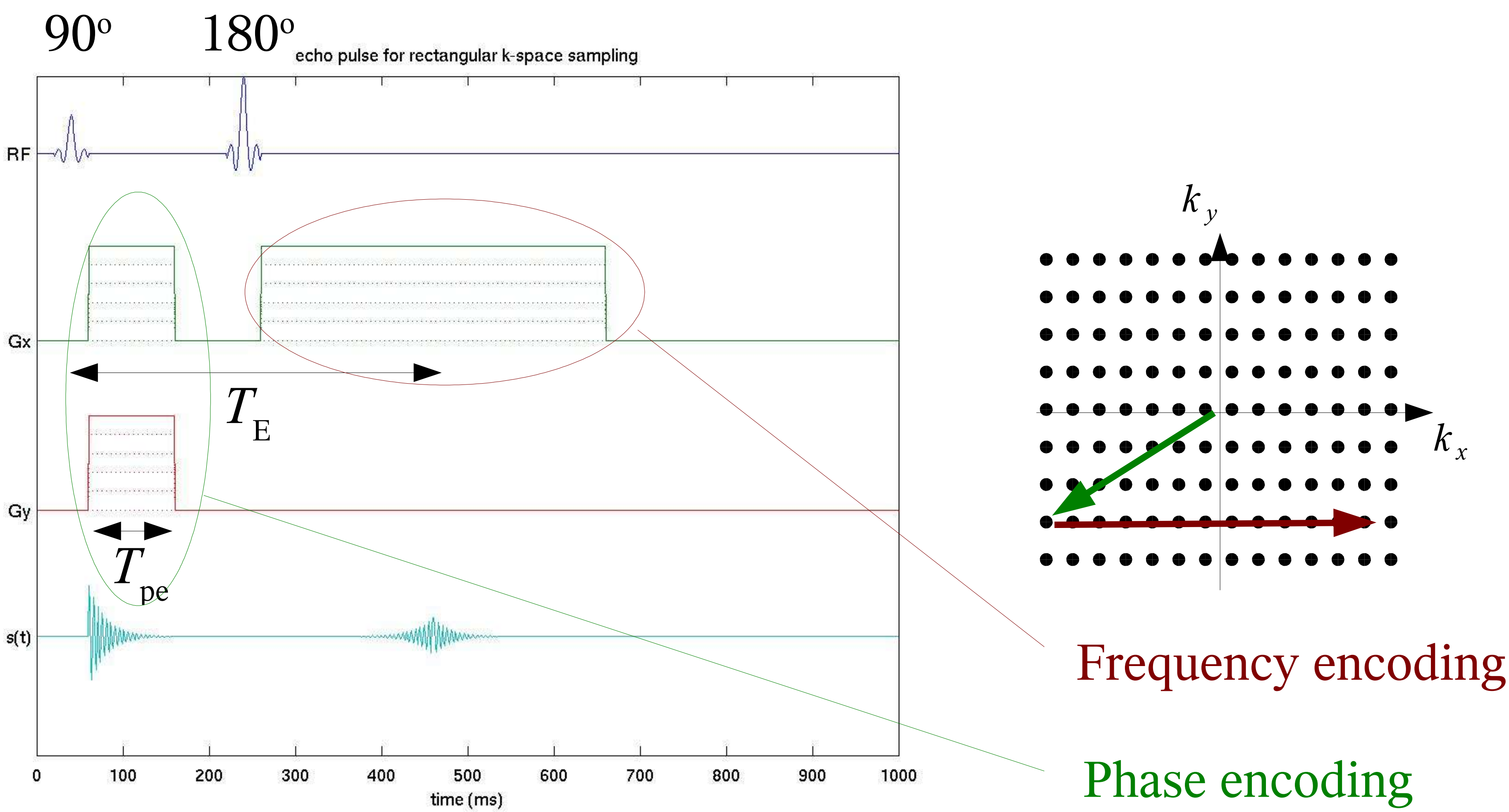
MRI – Phase Encoding

If spins precess at different speeds (due to a variable encoding gradient G_y) during a fixed amount of time T_{pe} they will gain a different phase:



MRI – Phase Encoding

A phase encoding echo pulse sequence, which will sample the k-space along the k_x axis for different values of k_y is as follows:



MRI – Phase Encoding

With a phase encoding gradient G_y in y direction and frequency encoding gradient in G_x in x direction the echo signal would be (ignoring the z direction again):

$$S(t, T_{pe}) = \int dx\, dy\, \rho(x, y)\, e^{-i\gamma G_x x (t - T_E) + \gamma G_y y T_{pe}}$$

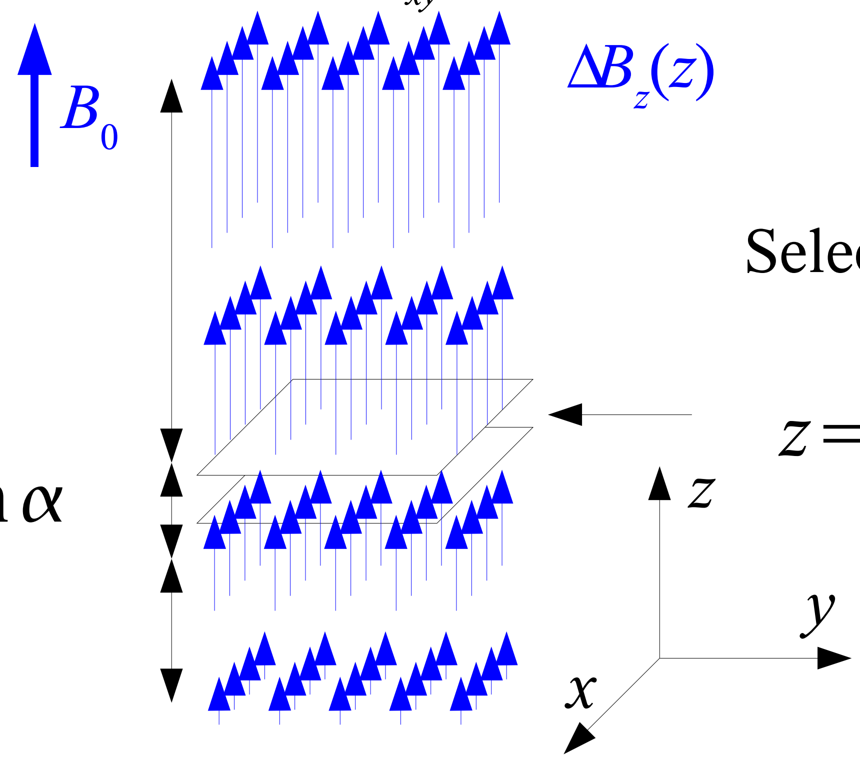
which can be rewritten as a 2D Fourier transform with the following definitions:

$$k_x = \gamma G_x (t - T_E) \qquad k_y = \gamma G_y T_{pe}$$

$$S(k_x, k_y) = \int dx\, dy\, \rho(x, y)\, e^{-i 2\pi (k_x x + k_y y)}$$

MRI – Slice selection

So far we considered gradients applied *after* the RF pulse during free precession. A gradient G_z *during* the RF pulse will select a transversal slice that satisfies the *resonance condition*: The RF pulse affects the spin precession coherently only if the frequency matches the B_z field. For the rest $M_{xy} = 0$ after α pulse.



$$M_{xy}(0)=0$$

$$M_{xy}(0)=M_0 \sin\alpha$$

$$M_{xy}(0)=0$$

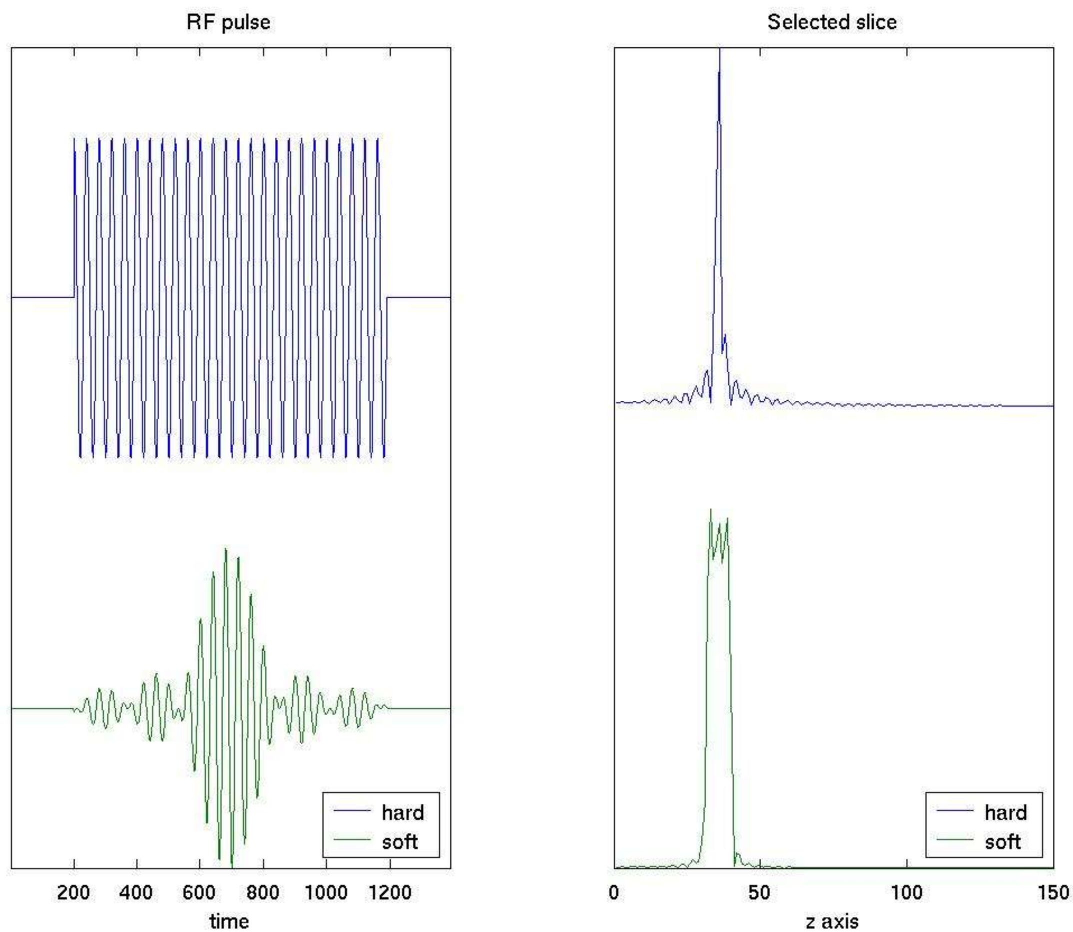
Selected slice at:

$$z=\frac{\gamma B_0-\omega}{\gamma G_z}$$

Only this slice will generate a signal!

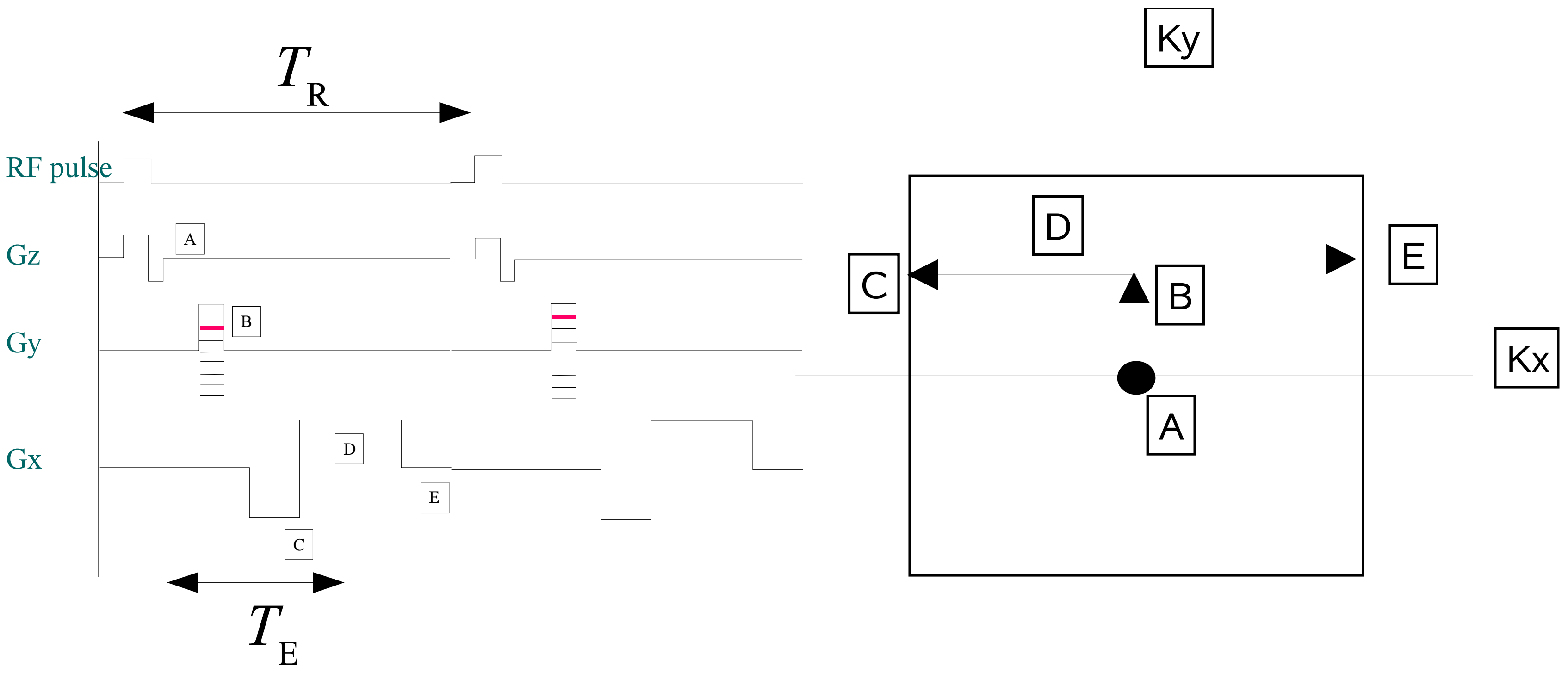
MRI – Slice selection

Note that a "hard" RF pulse contains high frequency components. It is therefore less selective in space as a "soft" pulse (sinusoid modulated by a sync functions - $\sin(\omega_0 t)*\mathrm{sinc}(\omega_1 t)$:



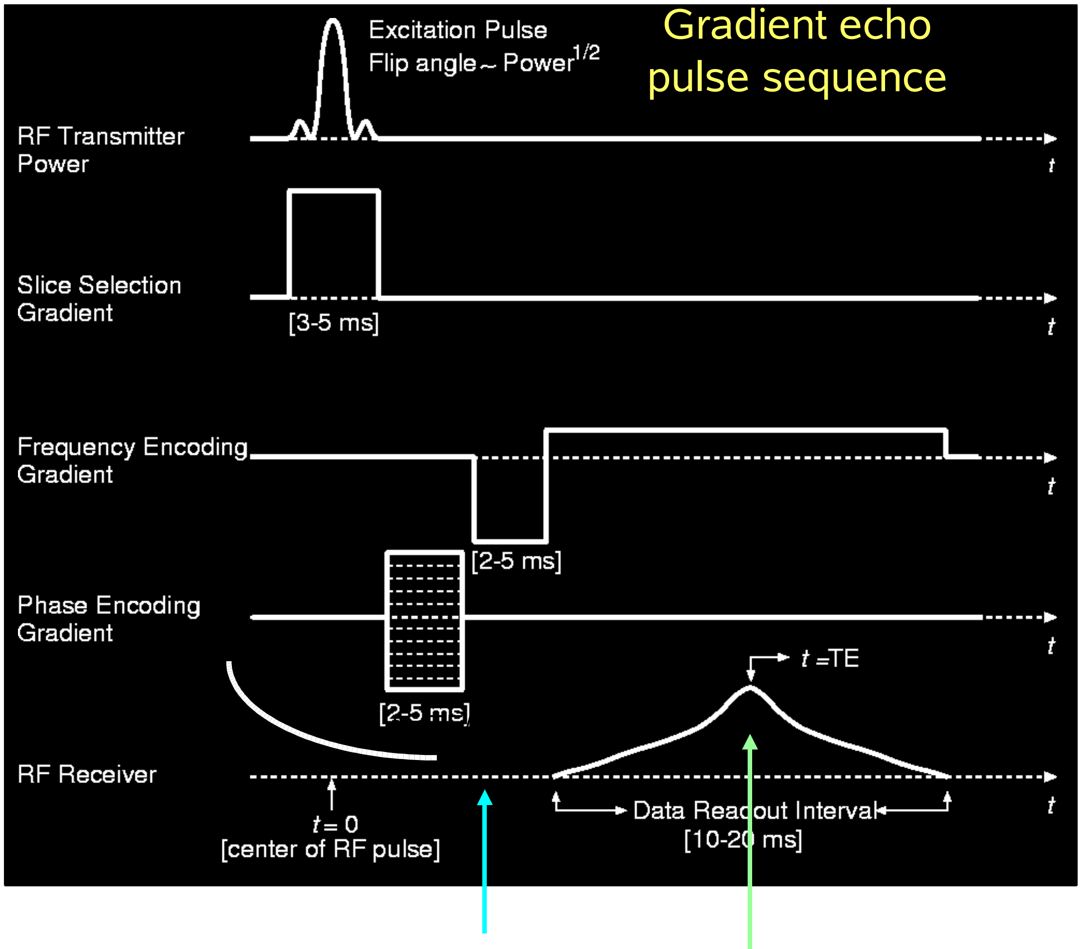
MRI – a pulse sequence example

Example for a full pulse sequence with gradient echo and the corresponding path in k-space:



Adapted from http://www.ecf.utoronto.ca/apsc/courses/bme595f/notes/

MRI – a pulse sequence example



A gradient reversal (shown) or 180 pulse (not shown) at this point will lead to a recovery of transverse magnetization

TE = time to wait to measure refocussed spins

Echos – refocussing of signal

Spin echo:

use a 180 degree pulse to “mirror image” the spins in the transverse plane

when “fast” regions get ahead in phase, make them go to the back and catch up

- measure T2
- ideally TE = average T2

Gradient echo:

flip the gradient from negative to positive

make “fast” regions become “slow” and vice-versa

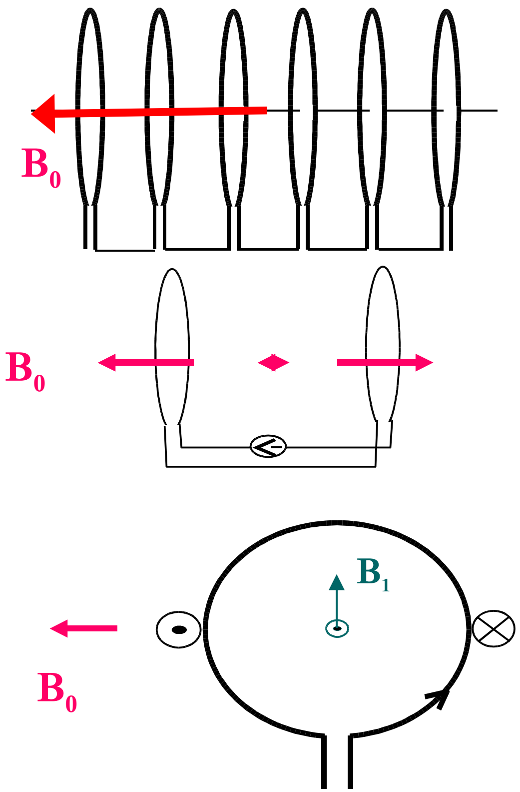
- measure T2*
- ideally TE ~ average T2*

Jody Culham, http://defiant.ssc.uwo.ca/Jody_web/fmri4dummies.htm

Source: Mark Cohen

MRI – Summary for Magnetic fields

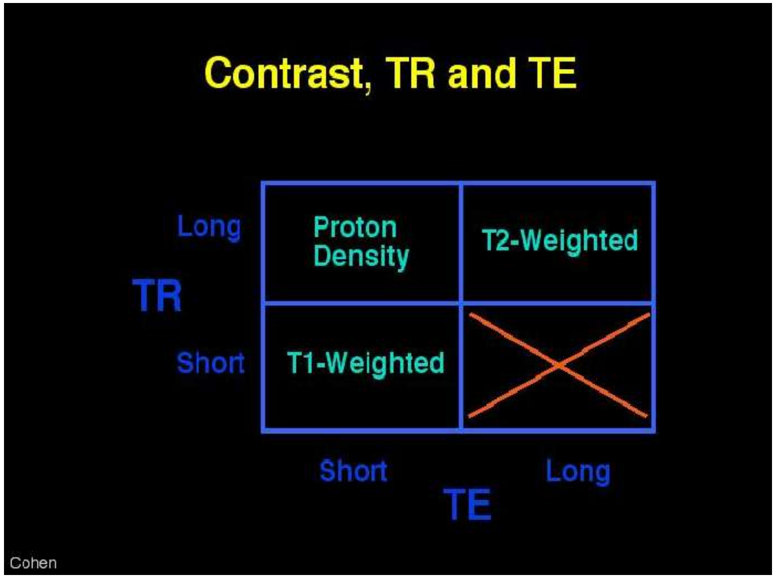
- Main Magnet
  - High, constant,Uniform Field, B_0.
- Gradient Coils
  - Produce pulsed, linear gradients in this field.
  - G_x, G_y, & G_z
- RF coils
  - Transmit: B1 Excites NMR signal (FID).
  - Receive: Senses FID.



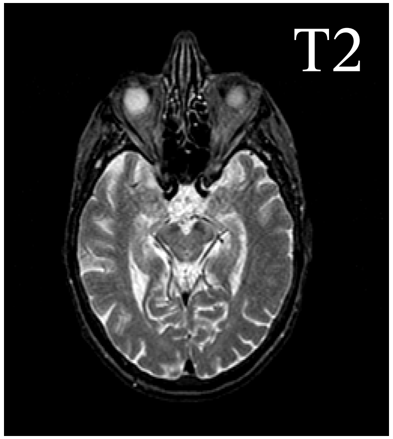
Adapted from http://www.ecf.utoronto.ca/apsc/courses/bme595f/notes/

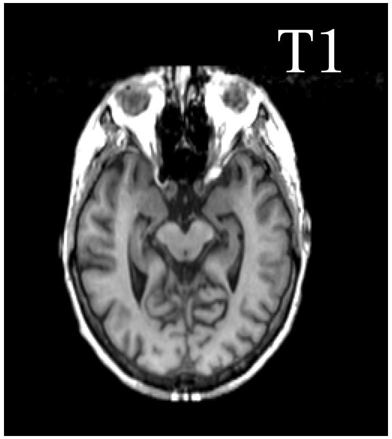
MRI – Contrast properties

- The strength of the NMR signal produced by precessing protons in a tissue depends on
  - T1, T2 of the tissue.
  - The density of protons in the tissue.
  - Motion of the protons (flow or diffusion).
  - The MRI pulse sequence used
- In a T1 "weighted" image the pulse sequence is chosen so that T1 has a larger effect than T2.
- Images can also be made to be T1, T2 proton density or flow/diffusion weighted.



Source: Mark Cohen





Adapted from http://www.ecf.utoronto.ca/apsc/courses/bme595f/notes/

MRI – Contrast, T1, T2

- MRI Contrast is created since different tissues have different T1 and T2.
- Gray Matter: (ms) T1= 810, T2= 101
- White Matter: (ms) T1= 680, T2= 92
- Bone and air are invisible.
- Fat and marrow are bright.
- CSF and muscle are dark.
- Blood vessels are bright.
- Gray matter is darker than white matter.